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Engineering Notes

Initial Orbit Design from Ground Track Points

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I. Introduction

MOTIVATED by the need for optimal orbit design algorithms for remote sensing space missions, the problem of orbit design given three ground sites on the ground track is addressed. Given three ground sites of interest, characterized by their right ascensions and latitudes, the objective is to calculate an orbit such that its ground track passes through the three sites within a given time frame. The problem has been formulated, and two solution algorithms have been developed. The first algorithm finds the exact orbit for which the ground track passes through the given ground sites; the second algorithm calculates an approximate solution. The approximate solution can be used as an initial guess in the first algorithm for an effective search for the exact solution. This note describes the case of three sites.

In previous developments [1], a heuristic approach was proposed to search for a natural orbit that enables the spacecraft to visit the n number of sites naturally, without the use of propulsion. This solution, if obtained, is a passive solution, because no propulsion is needed. The method worked for a small number of sites of interest. The success of this method depends on the locations of the sites and the available time frame [2].

In the case of too many sites of interest, the set of sites will be split into subsets, in an optimal sense. Toward developing a splitting algorithm, this note addresses the case of only three ground sites. For three arbitrary ground sites, what are the solutions (orbits) that have ground tracks visiting the sites within a given time frame? Visiting a ground site does not necessarily require that the satellite pass directly above the ground site. However, in this development, the satellite is required to pass directly above the given ground sites. The reason is that, in the case of visiting only three ground sites, there is always at least one orbit that passes directly above the three sites; hence, considering only these orbits limits the number of solutions and simplifies the analysis. In future developments, cases with higher numbers of ground sites will be considered, and the sensor's field of view will be taken into consideration to allow orbits with ground tracks not passing directly through the sites.

II. Problem Formulation

The right ascension, λ_i, and latitude, ϕ_i, for each ground site i are given at a given time.

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We seek an orbit such that the ground track of the spacecraft passes through the ground sites exactly within a given time frame. Two-body motion is assumed. In Fig. 1, in one Earth revolution about the spinning axis, each site intersects the orbit plane twice.

If, at the intersection time, the spacecraft position vector is not aligned with the position vector of the ground site, then the site is not on the ground track at that intersection (e.g., ground site j in Fig. 1). A site will be on the ground track if, and only if, at the intersection time, the spacecraft position vector is parallel to the site position vector, for example, ground site i in Fig. 1.

Therefore, the problem may be formulated as follows: given three ground sites and a time frame for visiting them, find an orbit such that each ground site position vector, at one of its intersections with the orbit plane, is parallel to the spacecraft position vector within the given time frame.

III. Accurate Algorithm

Assume that the spacecraft is initially above one of the ground sites, that is, one of the ground sites is initially in the orbit plane and its position vector is parallel to the spacecraft position vector. Let this be site 1. Let the true anomaly of the spacecraft at that time be φ_1. After a certain amount of time, another ground site, site 2, intersects the orbit plane and the spacecraft is above it at the intersection time. Let the true anomaly of the spacecraft at that time be φ_2. The amount of time the spacecraft travels from φ_1 to φ_2 is a function of the orbit semimajor axis, a, and eccentricity, e.

The independent variables in this problem are assumed to be the following four quantities: a, e, φ_1, and φ_2. Let the initial position vector of each ground site be $\mathbf{r}_{i0}$, $\forall\ i = 1\cdots 3$, whereas the position vector of each ground site at its intersection with the orbit plane is $\mathbf{r}_i$, $\forall\ i = 1\cdots 3$. Both vectors are expressed in the Earth-centered inertial frame. The difference between the two positions, $\mathbf{r}_{i0}$ and $\mathbf{r}_i$, is in the right ascension of the site i, which is changing at a rate equal to the Earth's angular velocity, ω_E, and can be calculated as a function of the elapsed time $(t_i^{S/C} - t_1^{S/C})$. Therefore, the position vector of ground site 2 at the intersection with the orbit plane, $\mathbf{r}_2$, can be calculated as follows:

$$\phi_{20} = \sin^{-1}(\mathbf{r}_{20}(3)/r_2), \quad r_2 = \| \mathbf{r}_{20} \|, \quad \lambda_{20} = \tan^{-1}(\mathbf{r}_{20}(2)/\mathbf{r}_{20}(1)) \tag{1}$$

$$\phi_2 = \phi_{20}, \qquad \lambda_2 = \lambda_{20} + \omega_E(t_2^{S/C} - t_1^{S/C}) \tag{2}$$

$$\mathbf{r}_2 = r_2[\cos(\phi_2)\cos(\lambda_2); \quad \cos(\phi_2)\sin(\lambda_2); \quad \sin(\phi_2)] \tag{3}$$

where ϕ_{20} and λ_{20} are the initial latitude and right ascension, respectively, of ground site 2, and ϕ_2 and λ_2 are the latitude and right ascension, respectively, of ground site 2 at the intersection with the orbit plane.

The time of spacecraft flight, $(t_2^{S/C} - t_1^{S/C})$, between sites 1 and 2 can be calculated in terms of the independent variables as follows [3]:

$$E_i = 2\tan^{-1}\left(\sqrt{(1-e)/(1+e)}\tan(\varphi_i/2)\right) \quad \text{and} \quad M_i = E_i - e\sin(E_i), \qquad i = 1, 2 \tag{4}$$

$$t_1^{S/C} = M_1\sqrt{a^3/\mu} \tag{5}$$

$$t_2^{S/C} = (M_2 + 2\pi k)\sqrt{a^3/\mu} \tag{6}$$

where k is an integer.

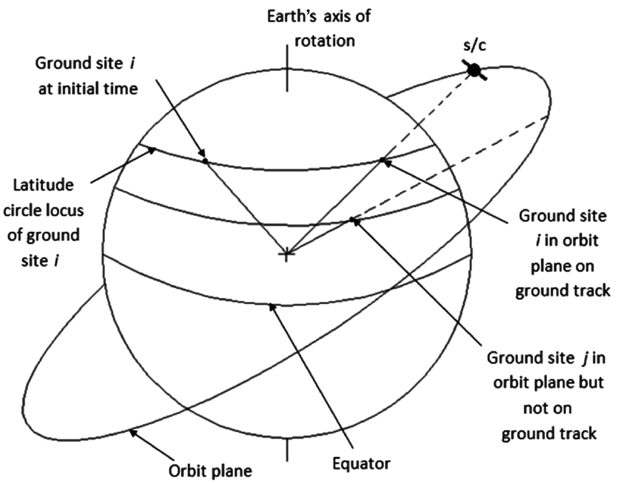

Fig. 1 Ground sites, initially not in the orbital plane, spin with the Earth and intersect the orbital plane at different times.

The two position vectors, $\mathbf{r}_1$ and $\mathbf{r}_2$, define the orbit plane. The position vector of the third ground site, $\mathbf{r}_3$, at the intersection with the orbit plane can be computed as a function of these two vectors. Two possible solutions exist for $\mathbf{r}_3$. Geometrically, these two solutions are the two intersections of the orbit plane with the circle locus of the ground site as Earth spins. Let these two vectors be $\mathbf{r}_{3j}$ where $j = 1$, 2. If the spacecraft's nadir is directly above ground site 3 at the time the vector $\mathbf{r}_{3j}$ is in the orbit plane, then the true anomaly of the spacecraft, φ_{3j}, can be calculated. The two vectors $\mathbf{r}_1$ and $\mathbf{r}_{3j}$ are known and are in the orbit plane. The angle between them is the difference between true anomalies $(\varphi_{3j} - \varphi_1)$ and can be calculated from the two vectors. There are two possible values of true anomaly for each position vector, φ_{3j-1} and φ_{3j-2}. Each one of the two true anomalies corresponds to a different orientation for the eccentricity vector, $\mathbf{e}$, with respect to the two position vectors $\mathbf{r}_{3j}$ and $\mathbf{r}_1$, as shown in Fig. 2. The two true anomalies are on two different orbits, with two opposite angular momentum vectors. For both cases, the times of flight of spacecraft since perigee, $t_{3j-k}^{S/C}$, associated with each true anomaly can be computed in terms of the eccentricity and semimajor axis. The right ascension of ground site 3 can be computed as follows:

$$\lambda_{3j} = \tan^{-1}(\mathbf{r}_{3j}(2)/\mathbf{r}_{3j}(1)) \tag{7}$$

Solving for the visiting time results in

$$\therefore t_{3j} = (\lambda_{3j} - \lambda_{30})/\omega_E + t_1 \tag{8}$$

If t_{3j} matches any of the $t_{3j-k}^{S/C}$ values, then the assumed orbit $(a, e, \varphi_1, \varphi_2)$ is the solution orbit to this configuration of ground sites.

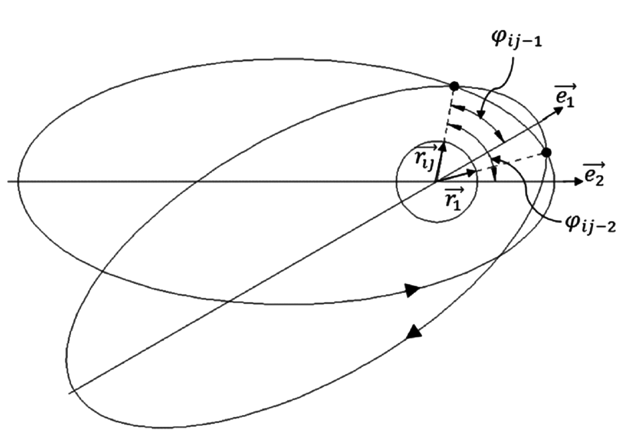

Fig. 2 Two possible orbit orientations in the orbit plane for given $\mathbf{r}_1$ and $\mathbf{r}_{ij}$.

To compare t_{3j} and $t_{3j-k}^{S/C}$, the former must be scaled to an orbital period scale.

Let the orbit period be τ. Given that τ is known as a function of the semimajor axis, we compute the modified t'_{3j} as

$$t'_{3j} = t_{3j} - \tau \mathrm{floor}(\mathrm{t_{3j}}/\tau) \tag{9}$$

where the floor function rounds toward the nearest integer toward minus infinity. Define the error function f_e as

$$f_e(a, e, \varphi_1, \varphi_2) = |t'_{3j} - t_{3j-k}^{S/C}| \tag{10}$$

The error function f_e has to be zero for an exact solution. The value of f_e for a certain orbit quantifies how far this orbit is from the exact solution orbit.

Let F_d be the mission duration, or mission time frame, in days. The first occurrence for a ground site i in the orbit plane is computed as already shown and t'_{3j} and $t_{3j-k}^{S/C}$ are evaluated. If the two times match, then the candidate orbit ground track passes through this ground site in its first orbit period. If this does not occur, then the times of subsequent occurrences for the ground site i in the orbit plane is $t_{3j} + M\tau_E$, where M is the number of days, and τ_E is the Earth's rotational period. Let $t_d = t'_{3j} - t_{3j-k}^{S/C}$. If the ground site is on the ground track, t_d must satisfy the following condition:

$$t_d = x - \mathrm{floor}(x) \tag{11}$$

where M is an integer, and

$$x = \frac{M(\tau_E - N\tau)}{\tau}, \qquad N = \mathrm{floor}(\tau_E/\tau), \quad \text{and} \quad M < F_d$$

IV. Approximate Algorithm

In this section, a simplified algorithm to obtain an approximate solution for the orbit design problem is described. This will let us create a smaller search space around that solution and use the earlier algorithm to obtain the exact solution. The algorithm described in Sec. III can be simplified by implementing the f and g function series approximation to estimate the orbit shape (semimajor axis and eccentricity).

The spacecraft is assumed to be initially above ground site 1, and $\hat{r}_1$ is assumed to be known. The independent design variables are selected to be the inclination, i, and the true anomaly, φ_1, of the spacecraft when ground site 1 is on the nadir. Also, we assume that i and φ_1 are known. Because $\hat{r}_1$ is in the orbit plane and the inclination is fixed, there are only two possible values for the right ascension of the ascending node, Ω. In general, to calculate the two possible values of Ω, the transformation of the vector $\mathbf{r}_1$ from the inertial frame to the perifocal frame [4] is used. Thus,

$$r_1[\cos(\varphi_1), \sin(\varphi_1), 0]^T = R_{313}(\Omega, i, \omega) r_1 \hat{r}_1 \tag{12}$$

where φ_1 is the true anomaly. Eliminating r_1 from both sides and considering only the third row results in

$$r_{1x} \sin(i) \sin(\Omega) - \sin(i) \cos(\Omega) r_{1y} + \cos(i) r_{1z} = 0 \tag{13}$$

The only unknown in Eq. (13) is Ω, and its solution yields the expected two values for Ω. Thus, we have two possible planes for the orbit: (i,Ω_1) and (i,Ω_2).

Next, the argument of perigee, ω, is calculated. Given that the orbit plane is fixed, and because the true anomaly of the spacecraft when it is above site 1 (φ_1) and the direction of spacecraft at that point $(\hat{r}_1)$ are known, there are two possible directions for the eccentricity vector in each orbit plane. These two directions are $(\pm\varphi)$ around $\hat{r}_1$ in the orbit plane, as shown in Fig. 2. From Fig. 2, the two possible directions for the $\hat{e}$ correspond to two different directions for the spacecraft motion. Because for each orbit plane the direction of spacecraft motion is fixed, one of the two values for ω is rejected; see Fig. 2. This leaves one possible value for ω in each orbit plane. The value of ω can be calculated using the second row of the vector equation (12). These types of equations have a closed-form solution, which is used to obtain the two pairs (Ω_1, ω_1) and (Ω_2, ω_2) as follows [5]:

$$r_{1x}\sin(i)\sin(\Omega) - \sin(i)\cos(\Omega)r_{1y} = R\cos(\Omega - \alpha) \tag{14}$$

$$R\cos(\Omega - \alpha) = -\cos(i)r_{1z} \tag{15}$$

where $R = \sqrt{(-r_{1z}\sin(i))^2 + (r_{1x}\sin(i))^2}$ and $\alpha = \frac{r_{1x}\sin(i)}{-r_{1z}\sin(i)}$. Therefore, it is now possible to solve for Ω using Eq. (15).

The unit vectors, $\hat{r}_2$ and $\hat{r}_3$, for spacecraft positions at the times of passage over sites 2 and 3 can be calculated from the eccentricity unit vector, $\hat{e}$, and $\hat{r}_1$. There are two possible solutions for each unit vector: $\hat{r}_{21}$, $\hat{r}_{22}$, $\hat{r}_{31}$, and $\hat{r}_{32}$. The time it takes the unit vector $\hat{r}_{i0}$ to rotate to $\hat{r}_{ij}$ is t_{ij} and can be calculated as follows:

$$t_{ij} - t_1 = \frac{(\lambda_{ij} - \lambda_{i0})}{\omega_E} \tag{16}$$

For each ground site i, each possible position vector $\mathbf{r}_{ij}$ corresponds to a true anomaly φ_{ij}, where $j = 1, 2$. This true anomaly can be calculated from φ_1, $\mathbf{r}_1$, and $\mathbf{r}_2$ as follows:

$$\varphi_{ij-1,2} = \varphi_1 \pm a\cos\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_{ij}}{r_1 r_{ij}}\right) \tag{17}$$

So far, we have two orbit planes and, in each plane, we have 16 possible combinations of φ_{ij-k}. In each case, the orbit shape (a and e) is calculated. Then the true solution(s) will be distinguished from the set of possible solutions. Let

$$\hat{r}_2 = c_1\hat{r}_1 + c_3\hat{r}_3 \tag{18}$$

where

$$c_1 = k_1 r_1/r_2 \quad \text{and} \quad c_3 = k_3 r_3/r_2 \tag{19}$$

The k_i coefficients are then approximated by implementing the f and g function series approximations as follows [6]:

$$\mathbf{r} = f_i\mathbf{r}_2 + g_i\mathbf{v}_2, \qquad i = 1, 3 \tag{20}$$

$$k_1 = \frac{\mathbf{r}_2 \times (f_3\mathbf{r}_2 + g_3\mathbf{v}_2)}{(f_1\mathbf{r}_2 + g_1\mathbf{v}_2) \times (f_3\mathbf{r}_2 + g_3\mathbf{v}_2)} = \frac{g_3}{f_1 g_3 - f_3 g_1} \tag{21}$$

$$k_3 = \frac{(f_1\mathbf{r}_2 + g_1\mathbf{v}_2) \times \mathbf{r}_2}{(f_1\mathbf{r}_2 + g_1\mathbf{v}_2) \times (f_3\mathbf{r}_2 + g_3\mathbf{v}_2)} = \frac{-g_1}{f_1 g_3 - f_3 g_1} \tag{22}$$

Using the series approximations for the f and g functions in Eqs. (21) and (22), we find that [6]

$$k_1 \approx a_1 + a_{1u}u \quad \text{and} \quad k_3 \approx a_3 + a_{3u}u \tag{23}$$

where

$$\tau_i = t_i - t_2, \qquad a_1 = \tau_3/(\tau_3 - \tau_1), \qquad a_3 = -(\tau_1/(\tau_3 - \tau_1)) \tag{24}$$

$$a_{1u} = \frac{\tau_3((\tau_3 - \tau_1)^2 - \tau_3^2)}{6(\tau_3 - \tau_1)}, \qquad a_{3u} = -\frac{\tau_1((\tau_3 - \tau_1)^2 - \tau_1^2)}{6(\tau_3 - \tau_1)} \tag{25}$$

Substituting the approximations for k_1 and k_3 into Eq. (19), c_1 and c_3 are found to be

$$c_1 = a_1(r_1/r_2) + a_{1u}(\mu/r_2^3)(r_1/r_2)$$
$$\text{and} \quad c_3 = a_3(r_3/r_2) + a_{3u}(\mu/r_2^3)(r_3/r_2) \tag{26}$$

Recalling the orbit equation for two-body motion, we can write $\alpha_i \equiv r_i/r_2 = (1 + e\cos(\varphi))/(1 + e\cos(\varphi_i))$, $i = 1, 3$. Substituting α_i into Eqs. (26), and eliminating $(1/r_2^3)$, we get

$$c_1 = \alpha_1[a_1 + (a_{1u}/a_{3u})((c_3/\alpha_3) - a_3)] \tag{27}$$

Both α_1 and α_3 in Eq. (27) are functions in eccentricity only. Therefore, Eq. (27) can be solved for e. The eccentricity can then be substituted into one of the two equations in (26) to obtain r_2. The semimajor axis, a, can then be calculated as

$$a = \frac{r_2(1 + e\cos\varphi_2)}{1 - e^2} \tag{28}$$

Equations (27) and (28) complete the orbit shape calculations (a and e) for each possible solution and, along with the corresponding i, ω, and Ω, identify a possible solution orbit. To evaluate whether the initial selection for the independent variables was accurate, the error function f'_e is calculated:

$$f'_e(i, \varphi_1) = \sum_{i=2}^{3} |t'_{ij} - t^{S/C}_{ij-k}|$$

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V. Numerical Example

As a numerical example, a set of three ground sites, arbitrarily selected, is considered. The three ground sites' right ascensions are [66.8757; 62.7950; 53.9206°] and their latitudes are [21.4690; 25.6589; 33.8258°], respectively. The time frame for visiting the three sites is selected to be 20 days. First the approximate algorithm is used to find an initial solution. The solution orbit has an inclination of 121.3° and the value of true anomaly when visiting the first site is 15.1°. The accurate algorithm is used with a range of inclinations from 116 to 128° and a range of φ_1 from 10 to 20°. The solution obtained passes directly above the given sites and has the following orbit parameters: $a = 16420.3$ km, $e = 0.0499$, $i = 120.02°$, $\omega = 10°$, and $\Omega = 79.997°$. In future developments, the $j2$ effect will be taken into consideration, and the design of repeated ground track orbits will be possible.

The regression of nodes is not negligible for the 20-day time frame and needs to be taken into consideration in orbit design, or a correction for the orbit will have to be performed. Regression of nodes is beyond the scope of this work. However, for natural orbits, 20 days is usually a wide time frame and solutions usually exist for shorter time frames.

In the two algorithms developed in this paper, the spacecraft was assumed to be above one ground site initially. Neither this assumption, or the selection of this site, affects the existence of the solution or the number of solutions. If this assumption is changed (by assuming that the satellite is not initially above any of the sites, or if we pick a different site to be initially on the nadir of the spacecraft), the same number of solutions is found. The new solutions will be the previous solutions, but rotated about the Earth's rotational axis.

VI. Conclusions

Two algorithms were developed to determine the orbit that visits a given set of points on the Earth's surface within a given time frame. The accurate algorithm uses the approximate solution as an initial guess and finds more accurate solutions in its neighborhood. There is always more than one solution for any three sites on Earth's surface in all of the tests conducted.

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